Mathe 1
Übungs-Bsp
27.03.2007

②

70 Welche Teilmenge der komplexen Zahlenebene beschreibt die angegebene Ungleichung?

$$\left|\frac{z+4}{z-4}\right| < 3$$

$$\frac{|z+4|}{|z-4|} < 3$$

$|z+4| < 3\,|z-4|$ $\qquad z+4 = (a+bi)+(4+0i) = (a+4)+bi$

$|(a+4)+bi| < 3\,|(a-4)+bi|$

$\sqrt{(a+4)^2+b^2} < 3\cdot\sqrt{(a-4)^2+b^2} \quad \Big|^2 \qquad$ LS, RS >0!

$(a+4)^2+b^2 < 9\cdot(a-4)^2 + 9b^2 \quad \Big| -9(a-4)^2 - b^2$

$(a+4)^2 - 9(a-4)^2 < 8b^2$

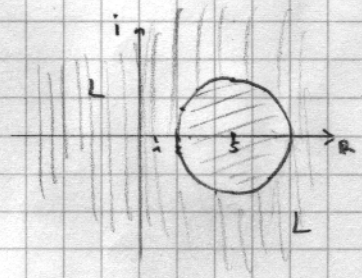
$a^2+8a+16-9(a^2-8a+16) < 8b^2$

$-8a^2+80a-128 < 8b^2 \quad |:8$

$-a^2+10a-16 < b^2$

$-a^2-b^2+10a-16 < 0 \quad |\cdot(-1)$

$a^2+b^2-10a+16 > 0$



Kreisgleichung! $\quad x^2+y^2+2mx+2ny+q=0 \qquad R=\sqrt{m^2+n^2-q}$, $x_0=-m$, $y_0=-n$

$a^2+b^2+2\cdot(-5)a+16=0$

$R=\sqrt{(-5)^2-16} = 3$ fix
$x_0 = 5$ fix
$y_0 = 0$ fix
da nur a, b variabel!

$|z| = r: \quad \frac{r+4}{r-3} < 3$

$r+4 < 3(r-4)$

$r+4 < 3r-12$

$16 < 2r$

$8 < r$

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73 Man berechne alle Werte von $\sqrt{8-6i} = a+bi$ ohne Benützung der trigonometrischen Darstellung. (Hinweis: Man quadriere die zu lösende Gleichung und vergleiche Real- und Imaginärteile)

$$\sqrt{8-6i} = a+bi \quad \Big|^2 \qquad \underline{a,b \in \mathbb{R}}$$

$$8-6i = (a+bi)^2$$

$$8-6i = a^2+2abi-b^2 = (a^2-b^2)+2abi$$

① $8 = (a^2-b^2)$ ② $-6i = 2abi \quad |:2i$

$\underline{b^2 = a^2-8}$ $-3 = ab \Rightarrow a = -\frac{3}{b}$

② in ①

$$b^2 = \left(\frac{3}{b}\right)^2 - 8$$

$$b^2 = \frac{9}{b^2} - 8 \quad |\cdot b^2$$

$$b^4 = -8b^2 + 9$$

$$b^4 + 8b^2 - 9 = 0$$

$$x_{1,2} = \frac{8}{2} \pm \sqrt{\left(\frac{8}{2}\right)^2 - 9}$$

$$b^2_{1,2} = \frac{8}{2} \pm \sqrt{\left(\frac{8}{2}\right)^2 + 9} = 4 \pm 5 = \{9, -1\}$$

a) $b_1^2 = 9 \Rightarrow b_1 = \pm 3 \longrightarrow$ in ② $a_1 = -\frac{3}{3} = -1$

b) $b_2^2 = -1 \Rightarrow b_2 \notin \mathbb{R}$ $a_2 = -\frac{3}{-3} = 1$

I: $\underline{3-i}$
II: $\underline{-3+i}$

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58 Man berechne ohne Taschenrechner alle Werte von

$\sqrt[4]{1+i}$ in der Form $[r, \varphi]$

$z = \sqrt[4]{1+i}$

$r = |z| = \sqrt{1^2 + 1^2} = \sqrt{2}$

$\arg(z) = \varphi = \arctan \frac{\overset{b}{1}}{\underset{a}{1}} = \frac{\pi}{4}$

$z = [\sqrt{2}, \frac{\pi}{4}]$

1. Lösung:

$z = [r, \varphi] \Rightarrow \sqrt[n]{z} = [\sqrt[n]{r}, \frac{\varphi}{n}]$

$\sqrt[4]{z} = [\sqrt[8]{2}, \frac{\pi}{16}] \left(= \sqrt[8]{2} \cdot \cos\left(\frac{\pi}{16}\right) + \sqrt[8]{2} \cdot \sin\left(\frac{\pi}{16}\right) \cdot i \right)$

$\sin\alpha, \cos\alpha = \sin(\alpha + 360°), \cos(\alpha + 360°) = \sin(\alpha + 720°), \cos(\alpha + 720°)$

Wurzel n-ten Grades hat n Lösungen:

$[\sqrt{2}, \frac{\pi}{4}] = [\sqrt{2}, \frac{\pi}{4} + 2\pi]$

① $[\sqrt[8]{2}, \frac{\pi}{16}]$

② $[\sqrt[8]{2}, \frac{\pi}{16} + 2\frac{\pi}{4}]$

③ $[\sqrt[8]{2}, \frac{\pi}{16} + 4\frac{\pi}{4}]$

④ $[\sqrt[8]{2}, \frac{\pi}{16} + 6\frac{\pi}{4}]$

63 Man beweise $\overline{\left(\frac{z_1}{z_2}\right)} = \frac{\overline{z_1}}{\overline{z_2}}$

$$\overline{\left(\frac{z_1}{z_2}\right)} = \overline{\left(\frac{a_1 + b_1 i}{a_2 + b_2 i}\right)} = \overline{\left(\frac{a_1 + b_1 i}{a_2 + b_2 i} \cdot \frac{a_2 - b_2 i}{a_2 - b_2 i}\right)} = \overline{\left(\frac{a_1 a_2 + b_1 b_2}{a_2^2 + b_2^2} + \frac{-a_1 b_2 + a_2 b_1}{a^2 + b^2} i\right)} =$$

$$= \frac{a_1 a_2 + b_1 b_2}{a_2^2 + b_2^2} - \frac{-a_1 b_2 + a_2 b_1}{a_2^2 + b_2^2} i = \underline{\underline{\frac{a_1 a_2 + b_1 b_2}{a_2^2 + b_2^2} + \frac{a_1 b_2 - a_2 b_1}{a_2^2 + b_2^2} i}}$$

$$\frac{\overline{z_1}}{\overline{z_2}} = \frac{a_1 - b_1 i}{a_2 - b_2 i} = \frac{a_1 - b_1 i}{a_2 - b_2 i} \cdot \frac{a_2 + b_2 i}{a_2 + b_2 i} = \underline{\underline{\frac{a_1 a_2 + b_1 b_2}{a_2^2 + b_2^2} + \frac{a_1 b_2 - a_2 b_1}{a_2^2 + b_2^2} i}}$$

QED

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⑤

56 Man bestimme rechnerisch (ohne Taschenrechner) und graphisch

Summe und Produkt der komplexen Zahlen

$$z_1 = 4+5i \qquad z_2 = \left[2, -\frac{\pi}{4}\right]$$

$z_2 = \left[2, -\frac{\pi}{4}\right]$

$$a = r \cdot \cos\varphi = 2 \cdot \cos\left(-\frac{\pi}{4}\right) = 2 \cdot \frac{\sqrt{2}}{2} = \sqrt{2}$$

$$b = r \cdot \sin\varphi = 2 \cdot \sin\left(\frac{\pi}{4}\right) = 2 \cdot \left(-\frac{\sqrt{2}}{2}\right) = -\sqrt{2}$$

$z_2 = \sqrt{2} - \sqrt{2}\, i$

$\underline{z_1 + z_2}$

$$(4+5i) + (\sqrt{2} - \sqrt{2}\, i) = \underline{(4+\sqrt{2}) + (5-\sqrt{2})\, i}$$

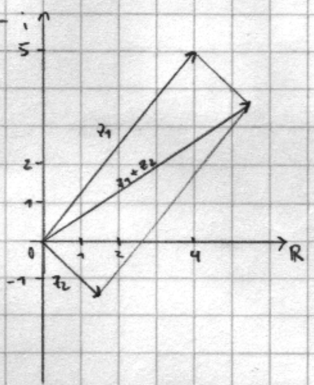
$\underline{z_1 \cdot z_2}$

$$(4+5i)\cdot(\sqrt{2} - \sqrt{2}\, i) = 4\sqrt{2} + 5\sqrt{2}\, i - 4\sqrt{2}\, i - 5\sqrt{2}\, i^2 =$$

$$= 9\sqrt{2} + \sqrt{2}\, i$$

$\underline{z_1 + z_2}$

$\sqrt{2} \approx 1{,}4$



$\underline{z_1 \cdot z_2}$